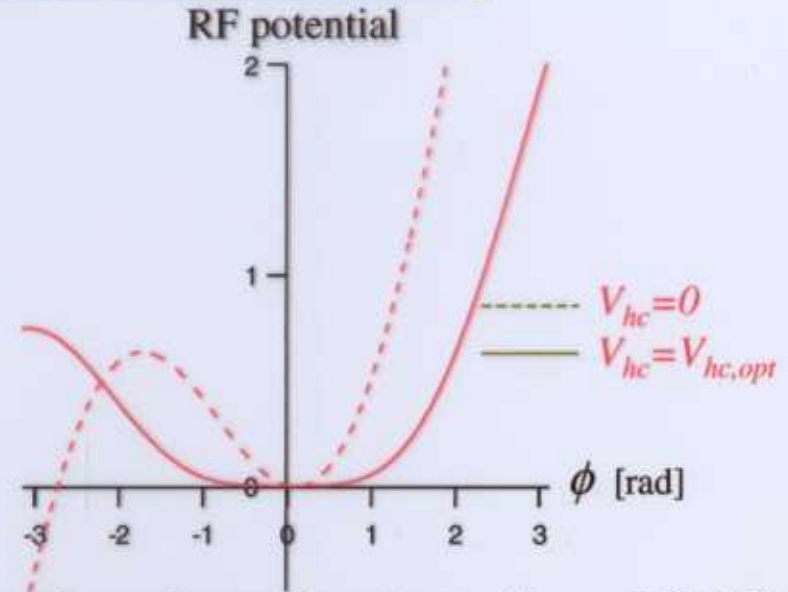
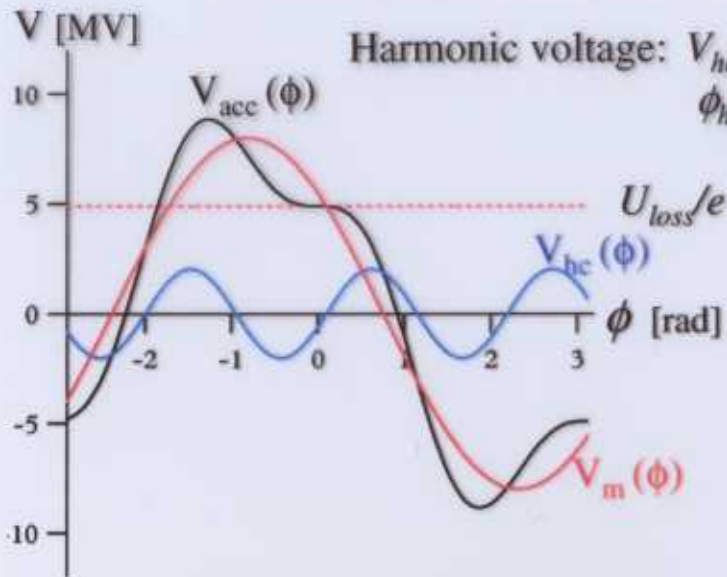


*Longitudinal Beam Dynamics
with Harmonic RF Systems*

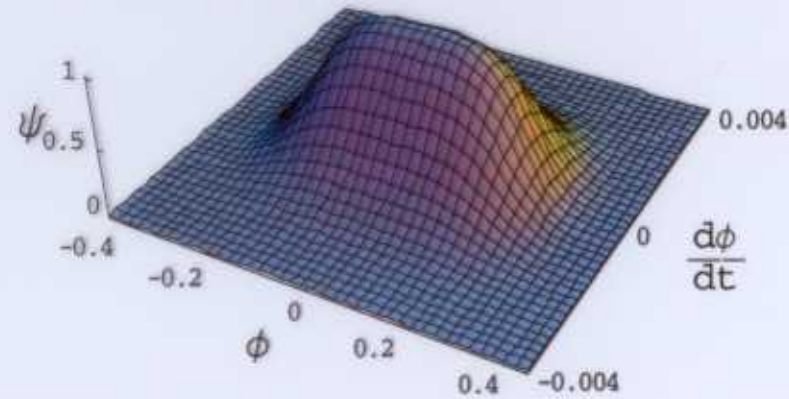
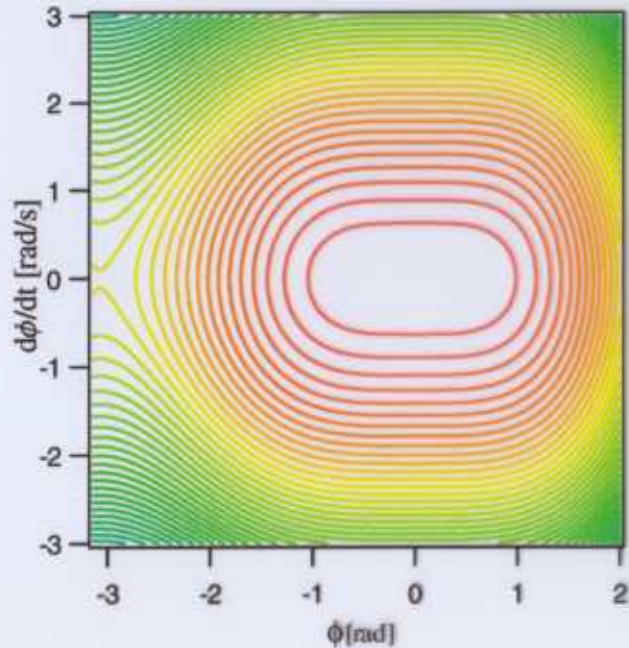
Vincent Serrière - ESRF

6th ESLS RF meeting, PSI, November 28 - 29, 2002

Double RF system - Optimum bunch lengthening case

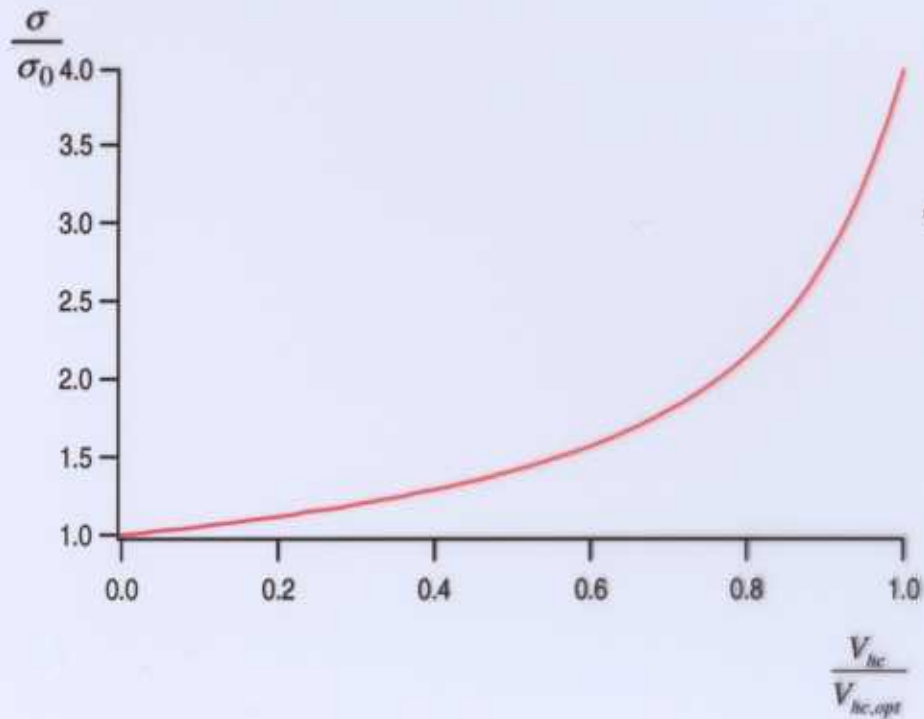


- optimum harmonic voltage : $V_{hc,opt} = 2.03$ MV
- RF potential: ϕ^2
- Non Gaussian longitudinal distribution

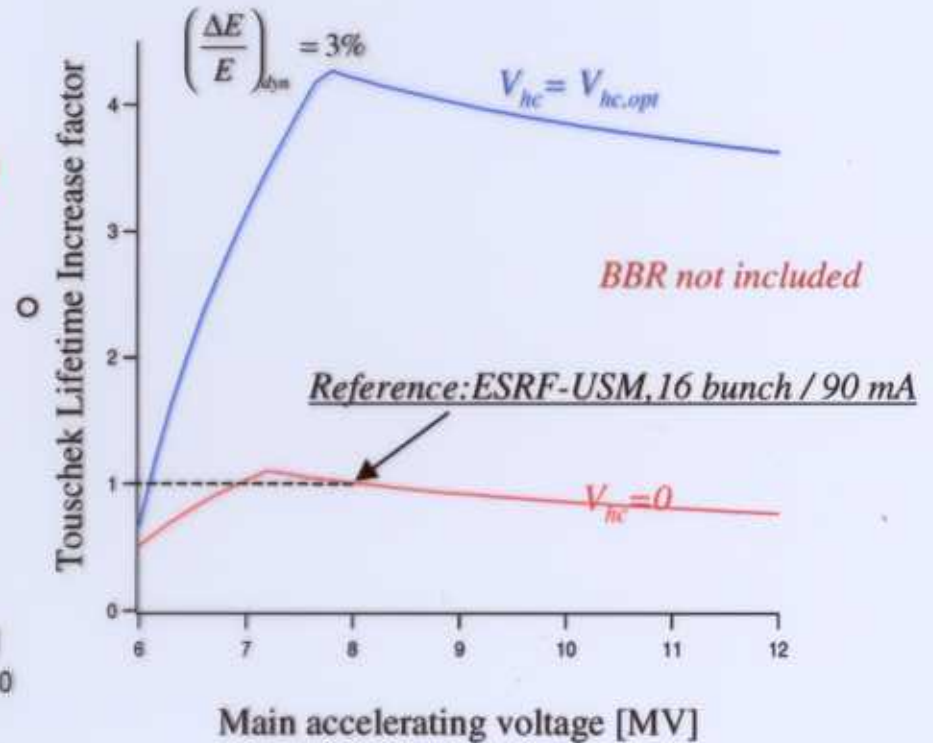


Bunch lengthening and Touschek lifetime improvement

ESRF parameters, $V_{acc.main} = 8 \text{ MV}$, main RF
only: $\sigma_0 = 16 \text{ ps}$



σ : rms "zero current" bunch length



Longitudinal Beam Dynamics

Synchrotron equation:
$$\ddot{\phi} + \frac{2\pi\alpha hf_o^2}{E_b} (e.V_{acc}(\phi) - U_o) = 0$$

with:
$$V_{acc}(\phi) = V_m \sin(\phi_s + \phi) + V_h \sin(n(\phi_h + \phi))$$

Beam motion:

Main RF system only:
$$\phi = \hat{\phi} \cos(2\pi f_{s0} t + \varphi)$$

Double RF system:
$$\phi = \hat{\phi} \text{cn}(\Omega(\hat{\phi})t, m)$$

Optimum bunch lengthening:
$$\begin{cases} m = 0.5 \\ \Omega(\hat{\phi}) = \frac{\pi^2}{K(0.5)} \sqrt{\frac{n^2 - 1}{12}} f_{s0} \hat{\phi} \end{cases}$$

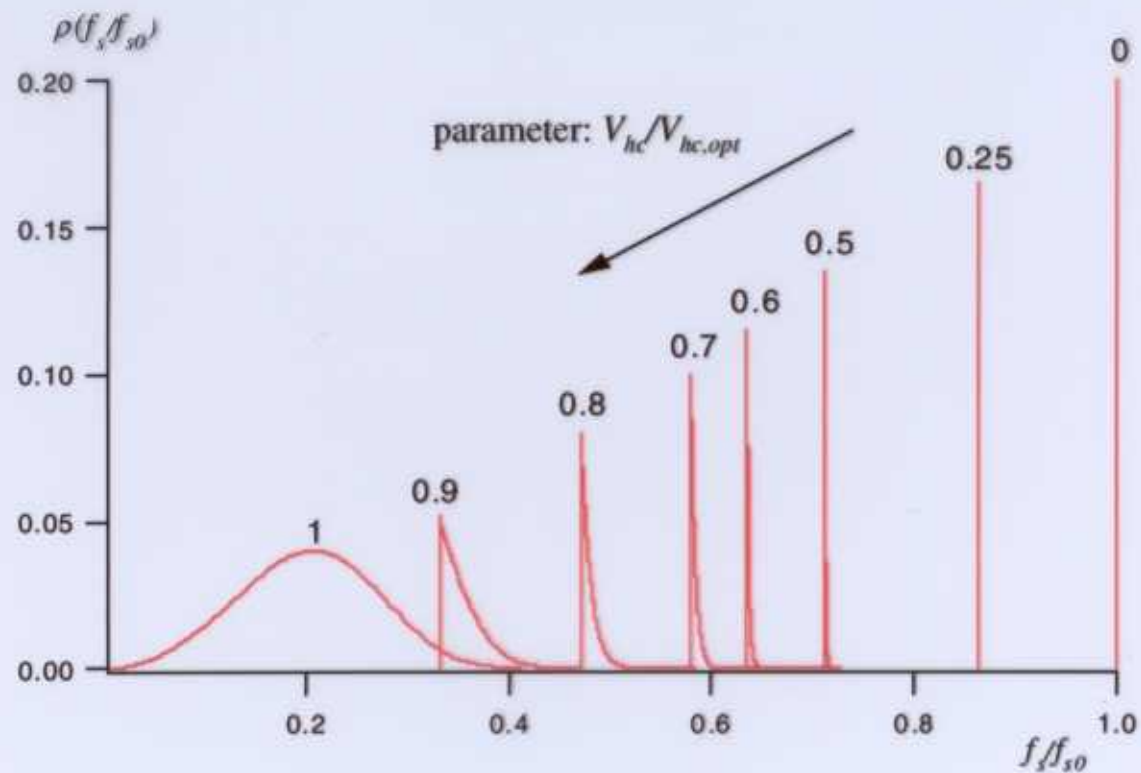
The synchrotron frequency Ω depends on the amplitude



Non linear beam motion

Synchrotron frequency distribution within bunches

The non linearity of the accelerating voltage induces a spread of the synchrotron frequencies within the bunches



ESRF parameters, third harmonic RF system

Touschek Lifetime Improvement at ESRF ?

Beam lifetime at the ESRF:	- Uniform filling:	$\xi_v = 0.4 \text{ to } 0.5$	200 mA	$\tau = 60 \text{ h}$
	- 16 bunch filling:	$\xi_v = 0.6$	90 mA	$\tau = 10 \text{ h}$
	- Single bunch:	$\xi_v = 0.9$	20 mA	$\tau = 4 \text{ h}$

Motivation for a harmonic cavity at the ESRF:

Improve the beam lifetime for the **single** and **16 bunch** operation modes

An accurate evaluation of a harmonic RF system should take into account:

- The non-linearity of the beam motion
- All longitudinal instability mechanisms

Longitudinal Instability Mechanisms

Robinson Instabilities:

Interaction of the beam and the fundamental impedance of the RF systems

DC : limitation for smaller machines but not for ESRF

AC: concerns both single and 16-bunch fill patterns

→ Determines the minimum number of harmonic cavities for stable operation

Potential Well & Microwave Instability:

Interaction of the beam and the longitudinal broad band impedance of the storage ring

→ Increase of the bunch length and energy spread with the beam intensity

Concerns both single and 16-bunch fill pattern

Longitudinal Coupled Bunch Instability:

Interaction of the beam and the Higher Order Modes of the RF systems

Concerns only the 16-bunch fill pattern for the ESRF

Extended Tracking Code

- **Single bunch multiparticle** model:

- BESAC, Potential Well and Microwave Instability

[G.Besnier, C.Limborg, T.Günzel]

ESRF

- **Multibunch single particle** model:

- Transient beam loading effects with harmonic RF system

[J.Byrd, S.De Santis, J.Jacob, V.Serriere]

ALS, ESRF

- **Multibunch multiparticle** model

[J.Jacob, V.Serriere]

Only way to take into account intrabunch nonlinear effects on multibunch oscillations

- Allows computing the longitudinal beam motion taking into account simultaneously:

- Impedance of the RF systems \Rightarrow Robinson instabilities
- BBR \Rightarrow Potential well distortion & Microwave instability
- Higher Order Modes \Rightarrow Longitudinal Coupled Bunch Instabilities

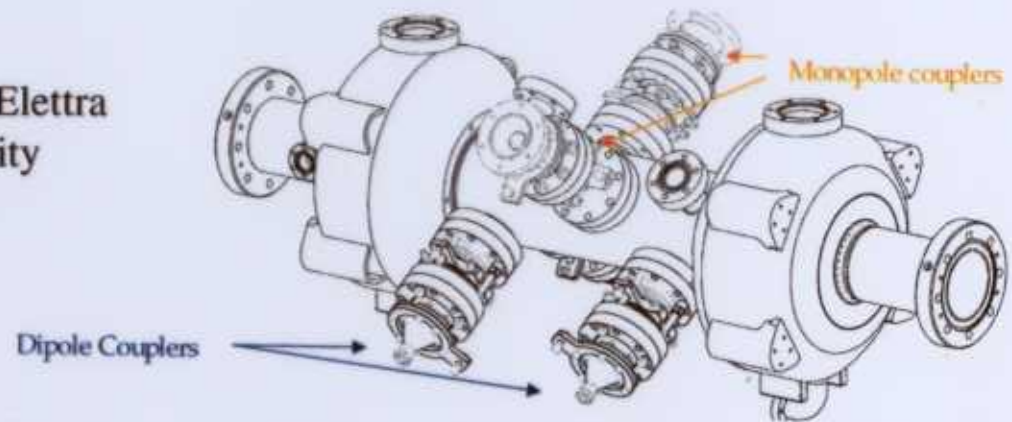
Harmonic Cavities for the ESRF ?

Only superconducting technology allows to provide the necessary optimum harmonic voltage within a reasonable space

Super-3HC cavity:

- 3rd harmonic cavity for: SLS & Elettra
- Scaling of the SC SOLEIL Cavity
- Construction: CEA & CERN

- $R_s/Q = 90 \Omega$
- Quality factor: $Q_0 = 2.10^8$
- $f_{res,hc} = 1.5 \text{ GHz}$



Superconducting Module with a pair of cavities

ESRF : Scaling of Super-3HC to $f_{res,hc} = 1056.6 \text{ MHz}$

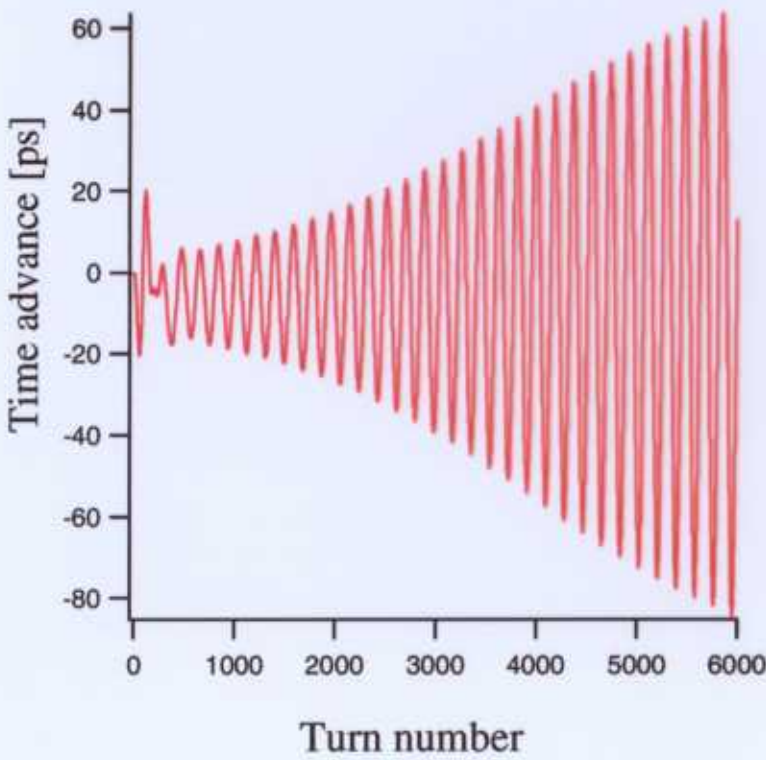
Passive harmonic RF system: the beam drives the harmonic voltage

Active harmonic RF system: the harmonic RF system is fed by an external generator

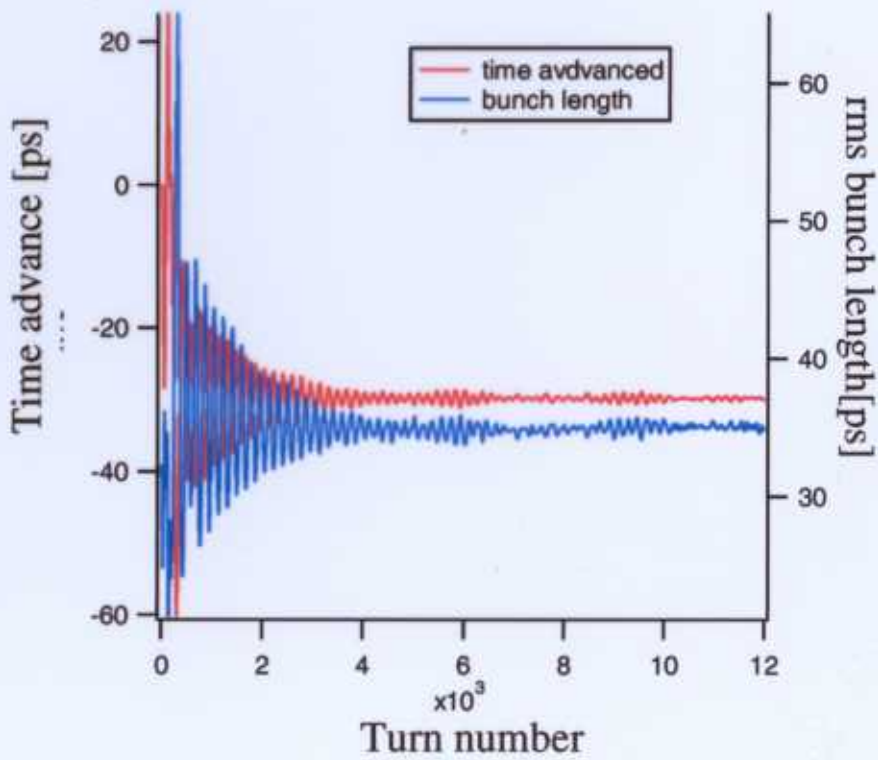
AC Robinson Instability for passive cavities @ ESRF

- $I_{beam} = 10\text{ mA}$ (single bunch)
- $V_{hc,set} = 1.5\text{ MV}$

2 modules/ 4 cells



4 modules/ 8 cells



Passive or Active Harmonic RF system for ESRF ?

Passive harmonic RF system:

- **4 modules / 8 cells necessary** for stable operation in single and 16-bunch operation modes.
- Even with a compact design a full ESRF straight section would be required.

Active harmonic RF system:

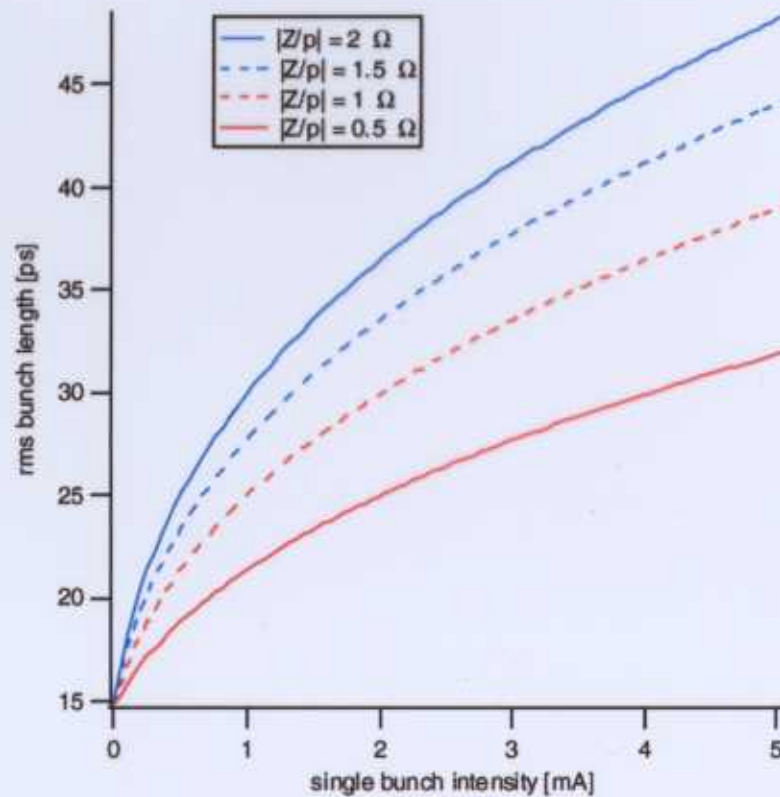
- **1 module / 2 cells sufficient.**
- Maximum generator power below 60 kW

Potential well

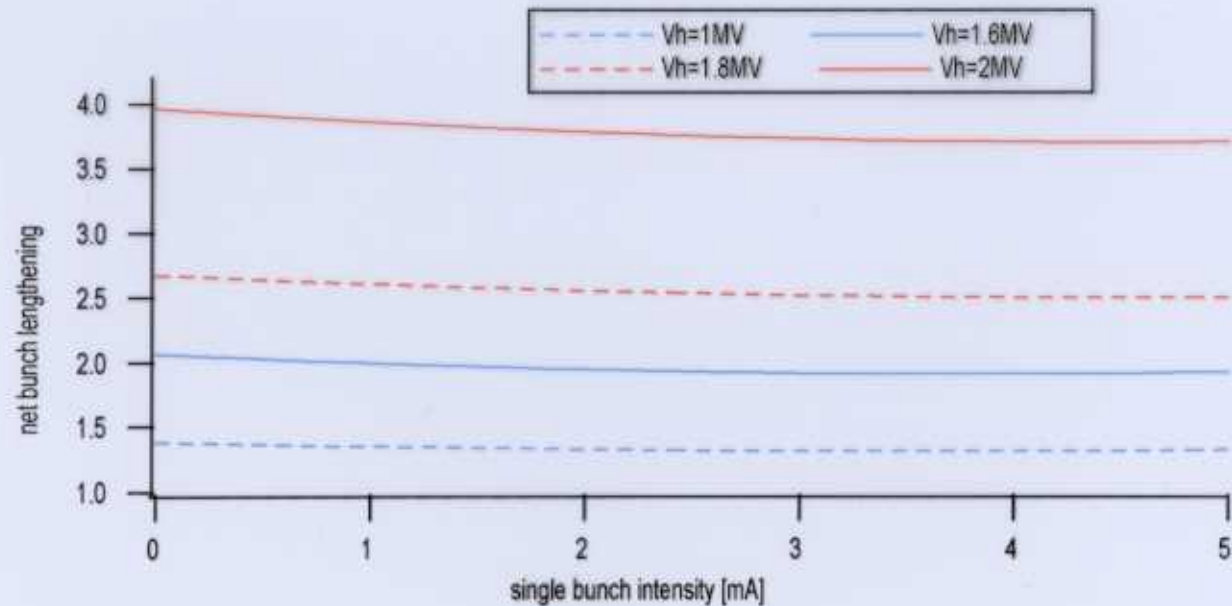
In the potential well regime, the longitudinal distribution is given by the Haissinski equation:

$$\Psi(\phi, \Delta E) = K_1 \Psi_0(\phi, \Delta E) \exp\left(-\frac{1}{2} \left(\frac{Q_s}{ah\sigma_E/E}\right)^2 \times h \left|\frac{Z_L \omega_0}{\omega}\right| (I(\phi) - I(0))\right)$$

Without harmonic RF system:



Potential well with harmonic RF system



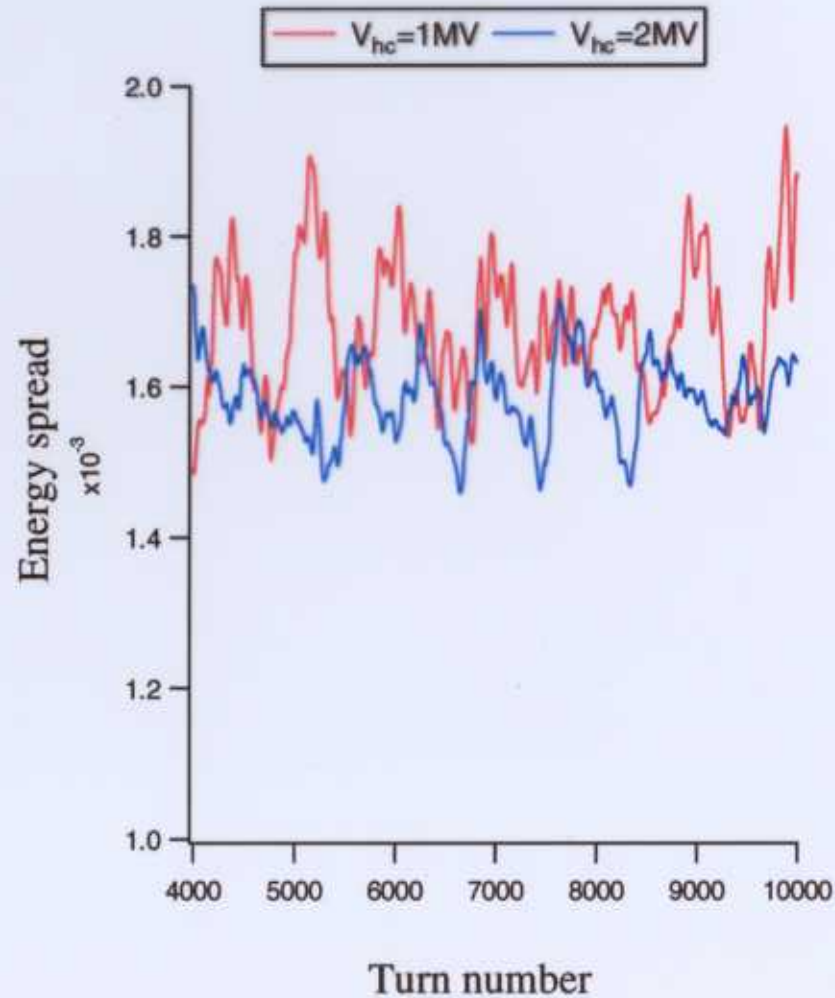
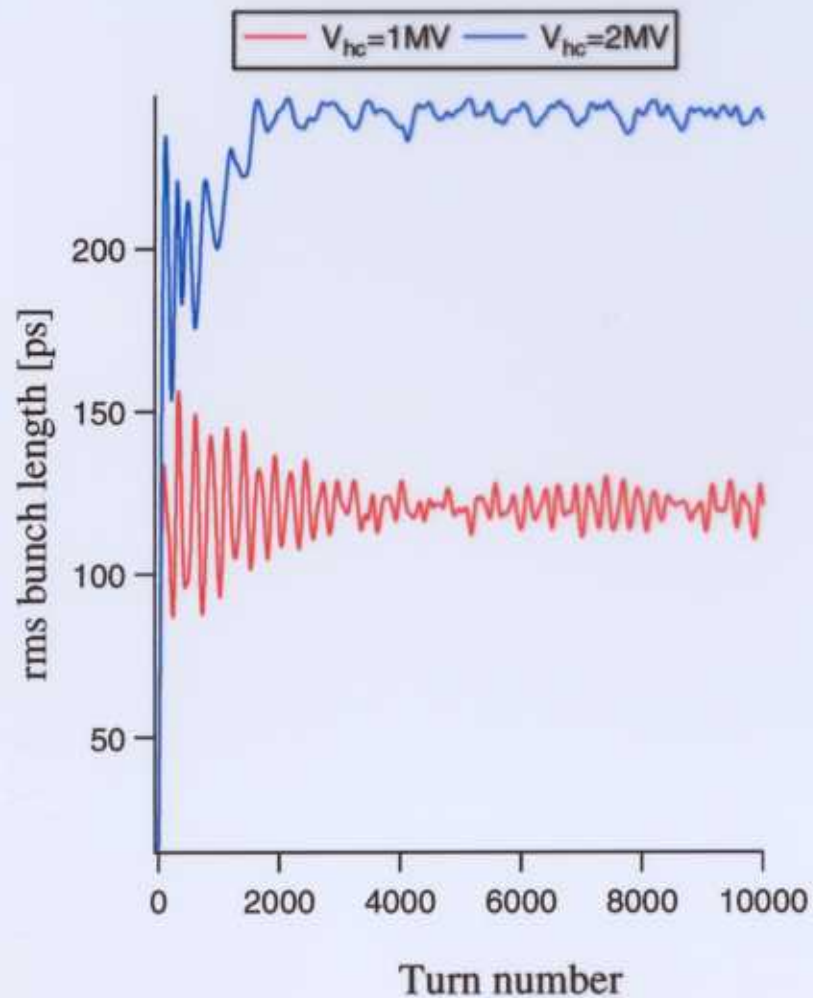
Tracking code & numerical resolution of the Haissinski equation:

the total bunch length increase is obtained by multiplying the elongations from the harmonic voltage and from the potential well effect

Microwave Instability - Numerical Example for ESRF -

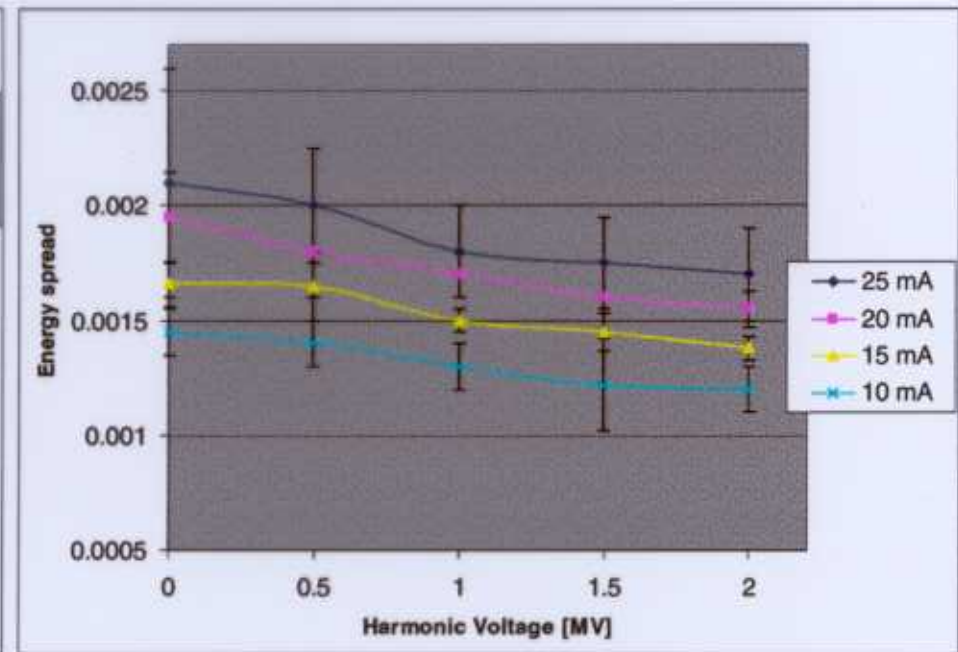
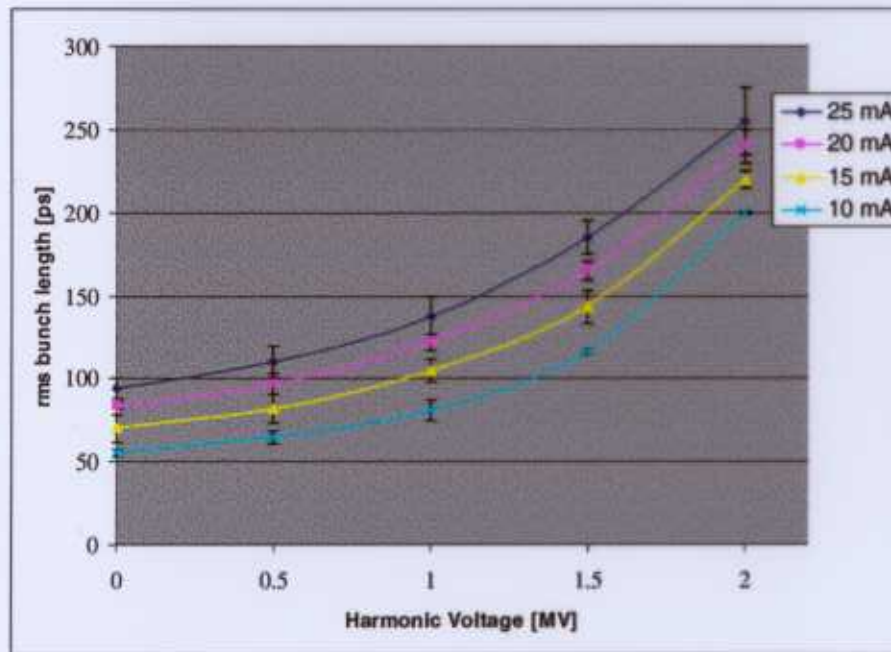
Standard ESRF BBR model: $f_{res} = 30 \text{ GHz}$, $R = 42 \text{ k}\Omega$, $Q = 1$

Single bunch - 20 mA



Microwave Instability bunch lengthening and energy spread @ ESRF

Single bunch operation



At 25 mA, a bunch length increase factor up to 2.7 can still be reached

A harmonic RF system reduces the increase in energy spread

Longitudinal Coupled Bunch Instability - typical ESRF case

NB: 4 bunches were sufficient to simulate LCBI for any symmetric fill pattern as uniform and 16 bunch operation mode

Main RF system only

$$I_b = 90 \text{ mA}$$

$$f_{res,HOM} = f_{RF} + 417 * f_0 + f_{s0}$$

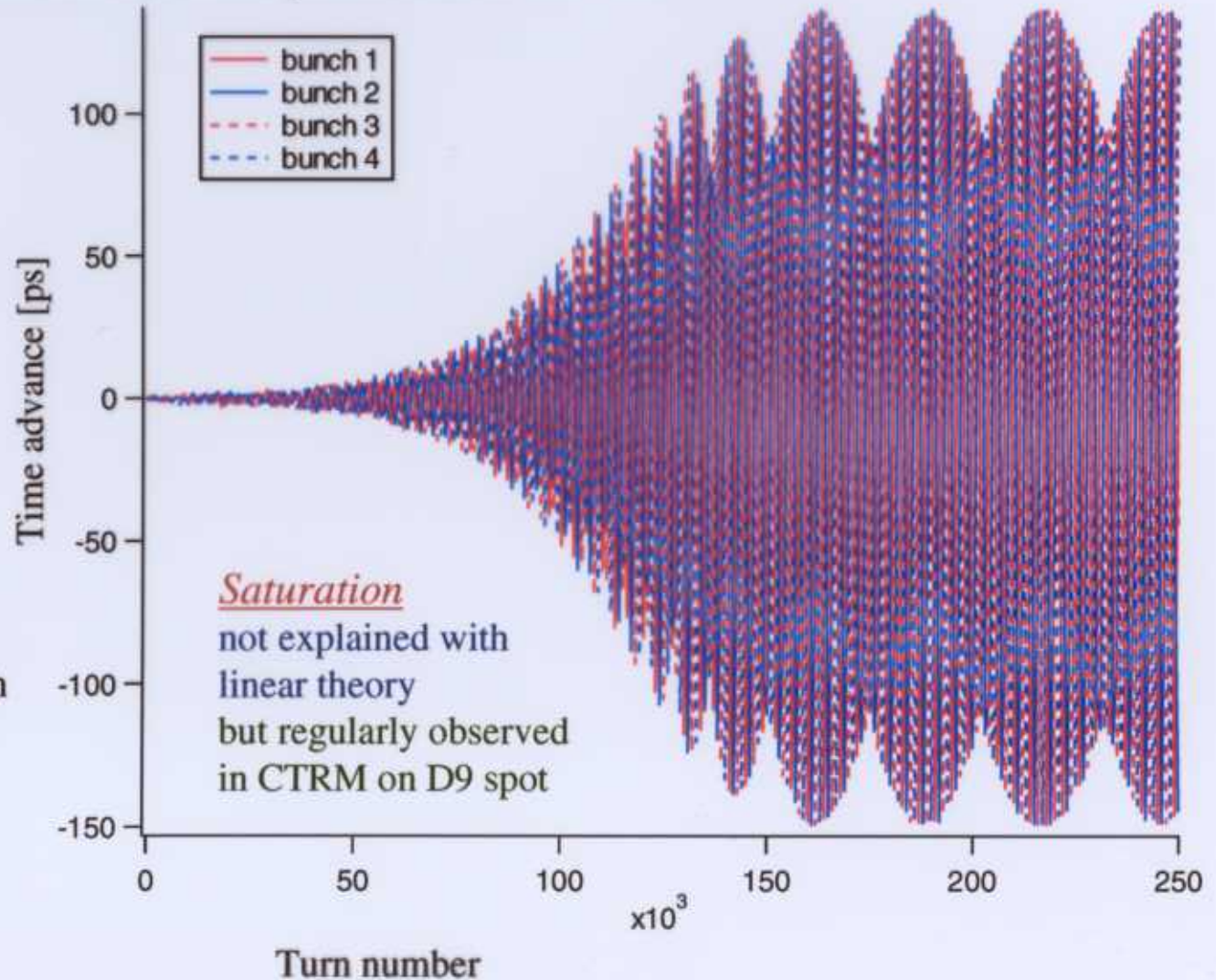
$$R_s = 2.2 \text{ M}\Omega \text{ \& } Q = 30000$$

Theoretical threshold:

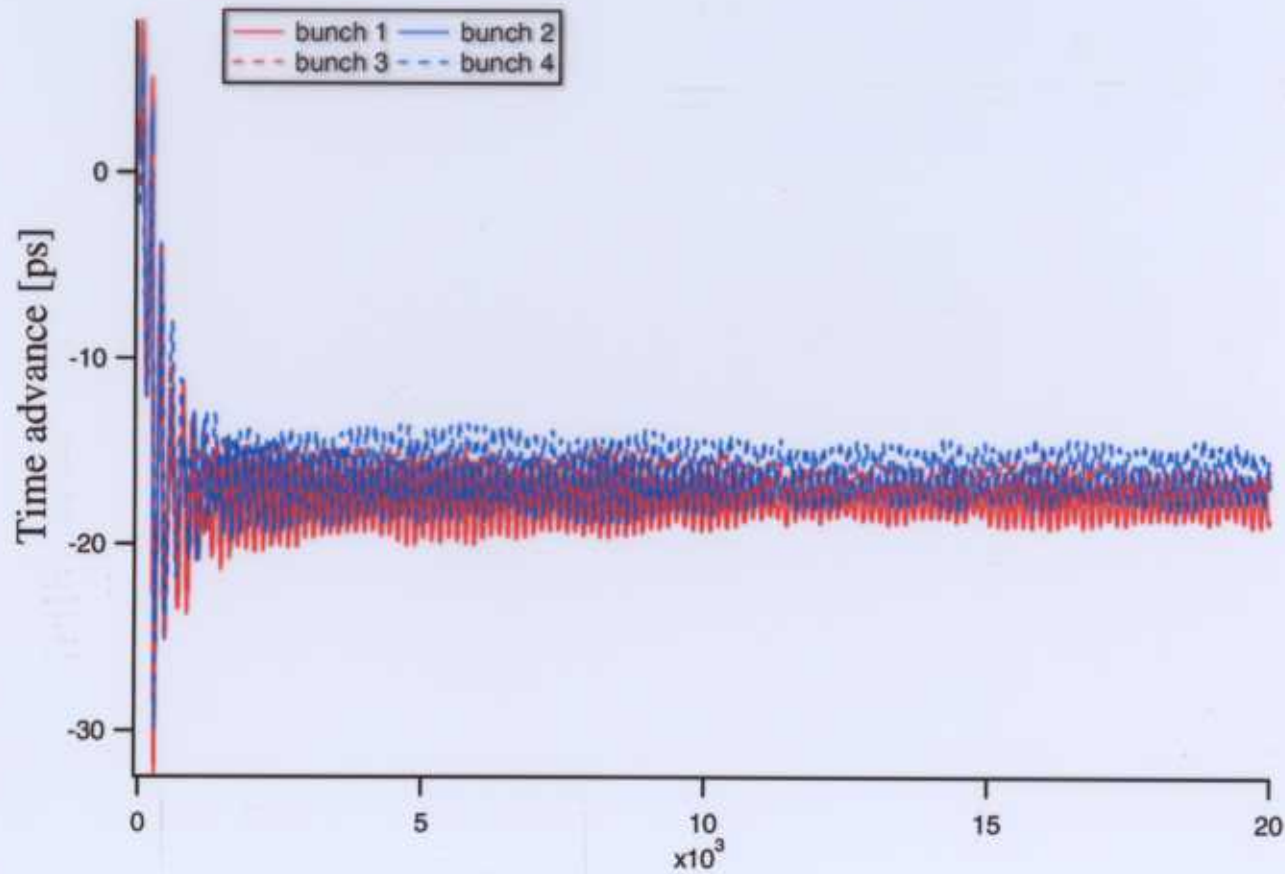
$$I_{th} = \frac{2 \delta_n f_{s0} T_0 E / e}{\omega_{HOM} R_{s,HOM}} = 91 \text{ mA}$$

The tracking code shows a modulation of the bunch length and the energy spread at $2 f_s$

$$\langle \sigma_E / E \rangle \approx 5 \times (\sigma_E / E)_0$$



Longitudinal Coupled Bunch Instability and Landau Damping with Harmonic RF system @ ESRF



$$I_{beam} = 95 \text{ mA}$$

$$V_{hc} = 1.6 \text{ MV}$$

$$f_{res,HOM} = f_{rf} + 417 * f_0 + \langle \rho(f_s, V_{hc}) \rangle$$

Turn number

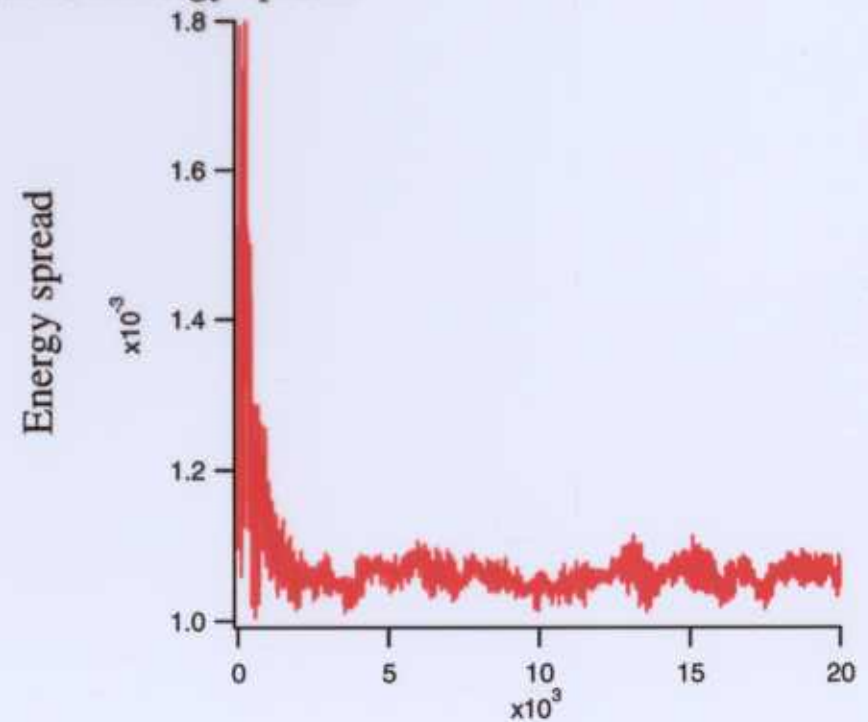
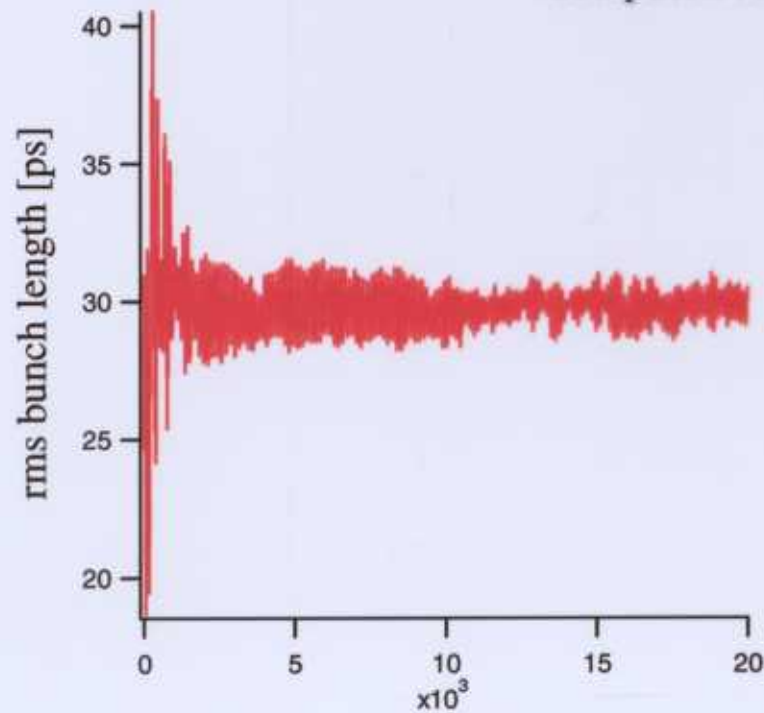
Longitudinal Coupled Bunch Instability and Landau Damping with Harmonic RF system @ ESRF

$$I_{beam} = 95 \text{ mA}$$

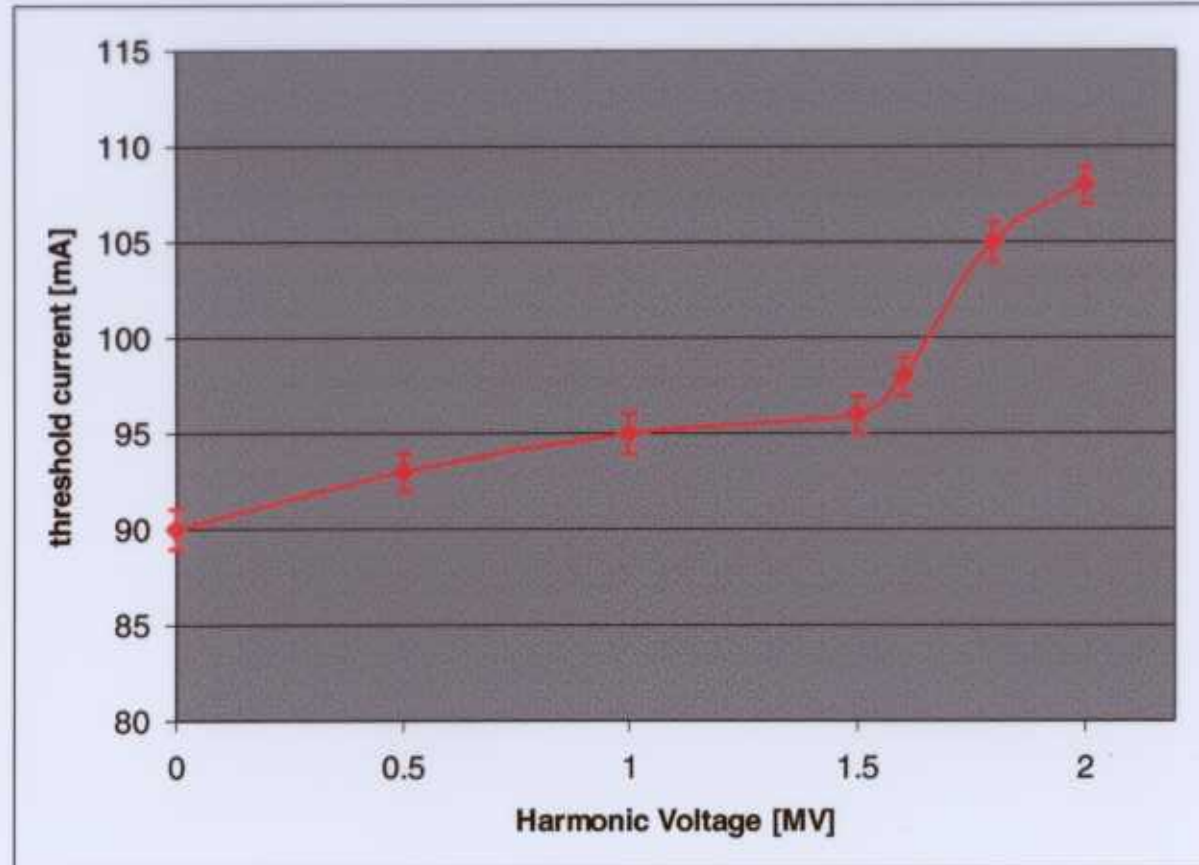
$$V_{hc} = 1.6 \text{ MV}$$

$$f_{res.HOM} = f_{rf} + 417 * f_0 + \langle \rho(f_s V_{hc}) \rangle$$

Computed bunch length and energy spread

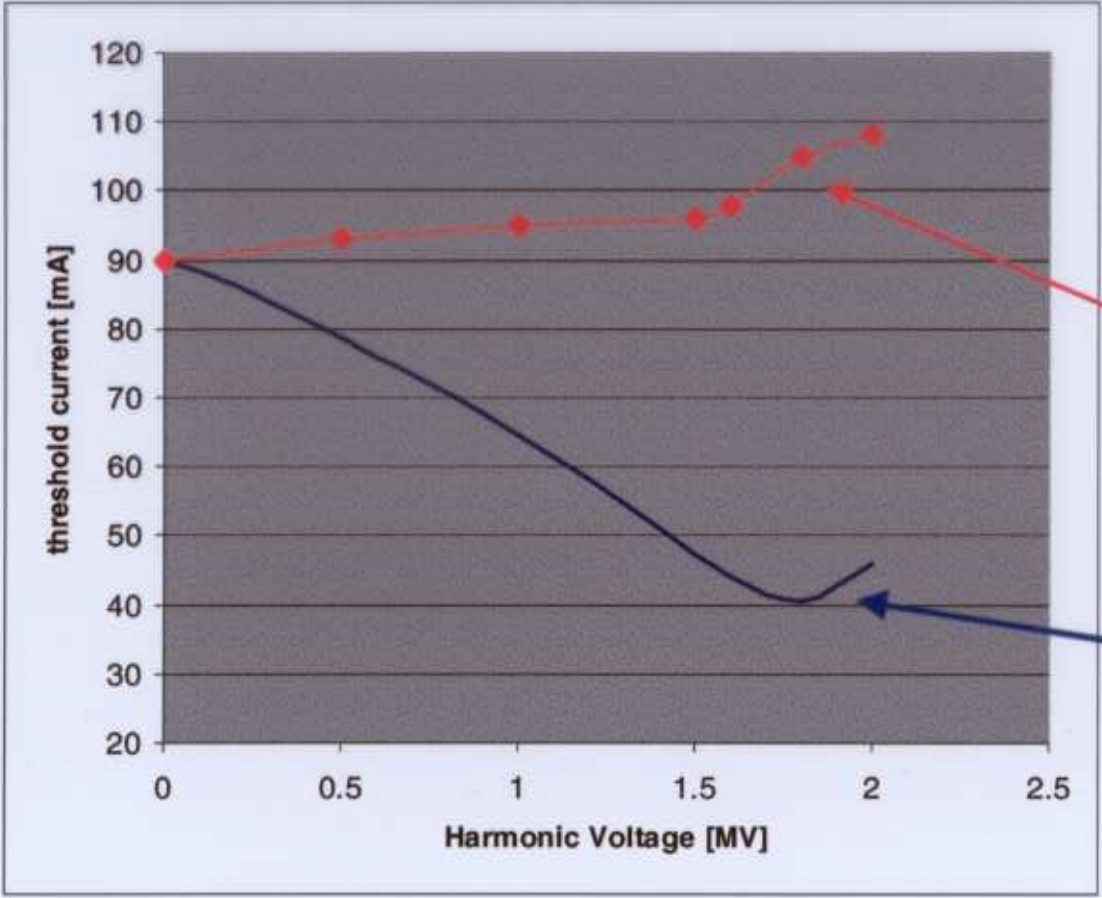


Longitudinal Coupled Bunch Instability Thresholds with a Harmonic RF system @ ESRF



Inside the marker the energy spread is increased by a factor up to 3

Longitudinal Coupled Bunch Instability Thresholds with a Harmonic RF system @ ESRF

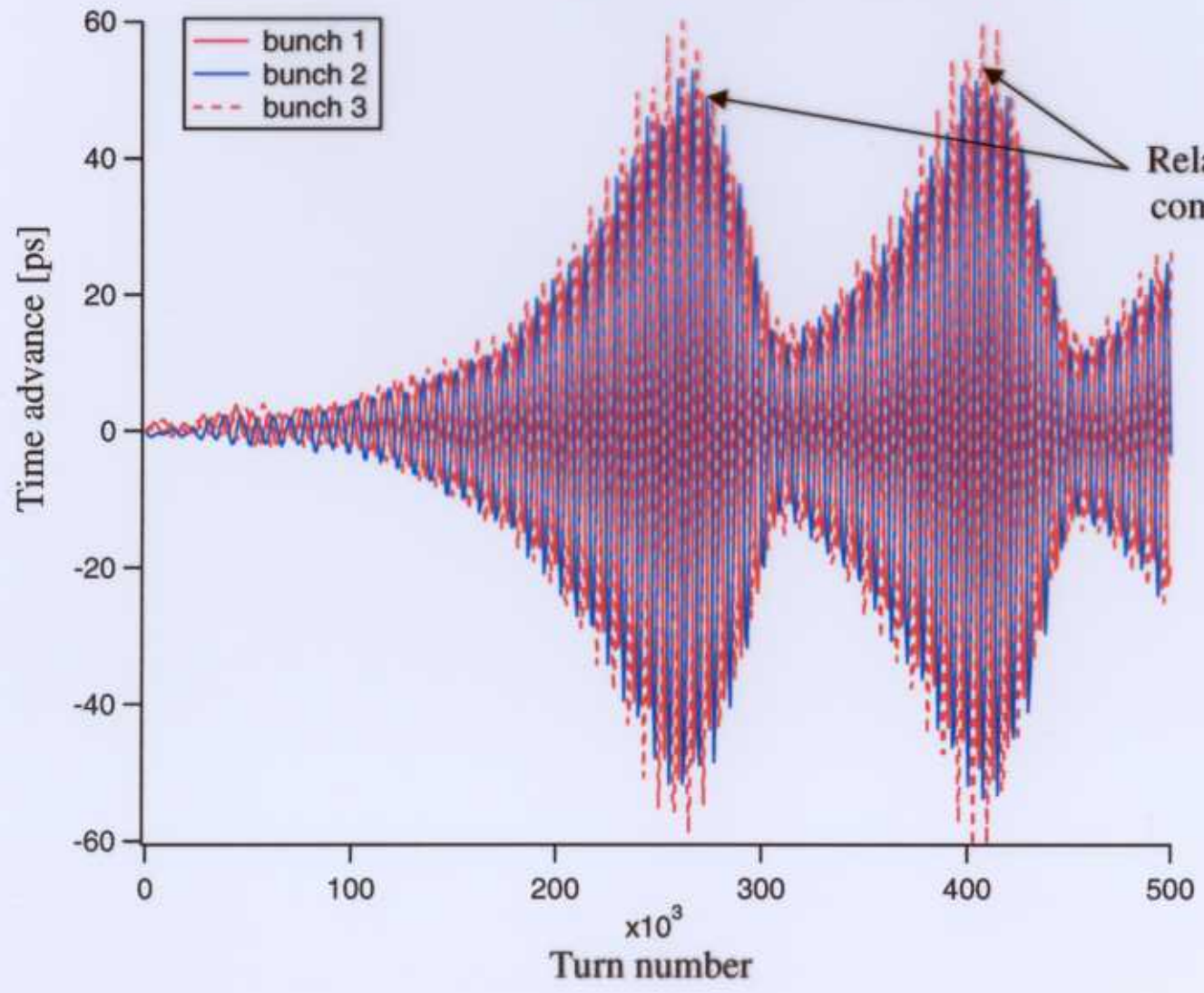


New tracking code

Linear model

Longitudinal Coupled Bunch Instability - Example of MAX-II

MAXII parameters
3 bunches and an arbitrary HOM



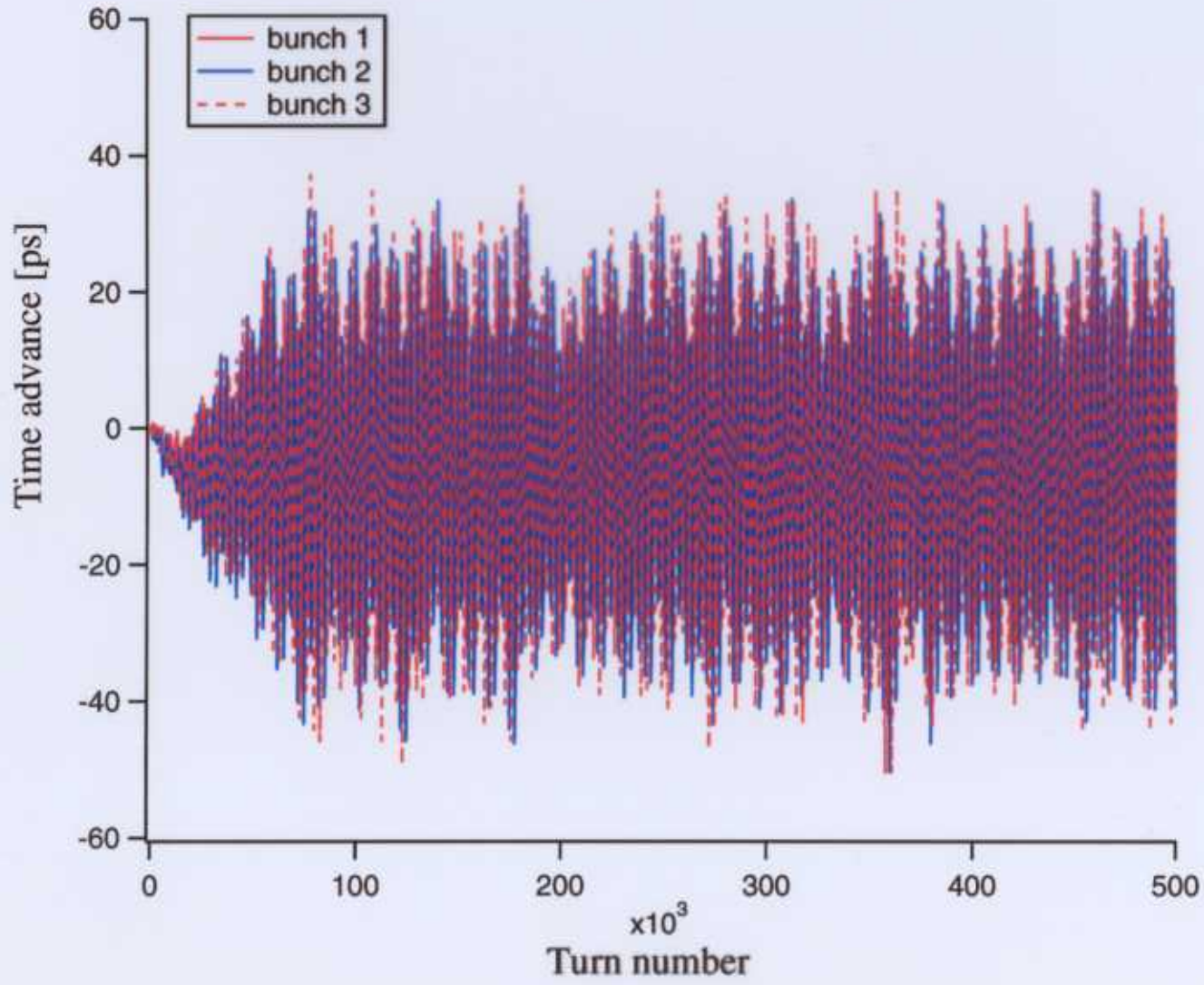
Relaxation also observed and computed by C.Limborg and J.Sebek / SPEAR

$$V_{hc} = 0$$

↓

$$\frac{\sigma_E}{E} \approx 1.1 - 2.3 \times 10^{-3}$$

LCBI - Example of MAX-II

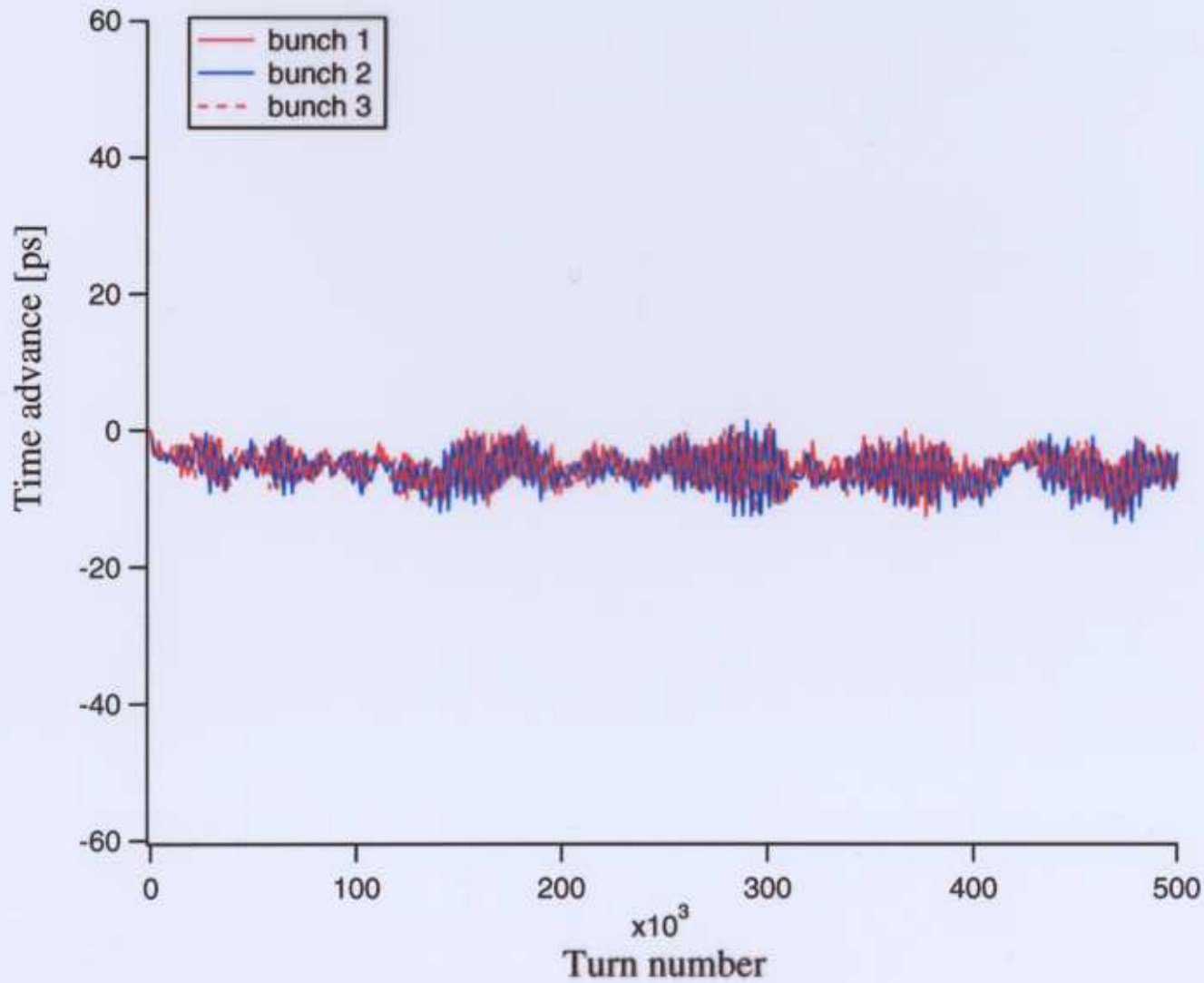


$$V_{hc} = 40 \text{ kV}$$

↓

$$\frac{\sigma_E}{E} \approx 1 - 1.6 \times 10^{-3}$$

LCBI - Example of MAX-II



$$V_{hc} = 80 \text{ kV}$$



$$\frac{\sigma_E}{E} \approx 7 \times 10^{-4}$$

(natural energy spread)

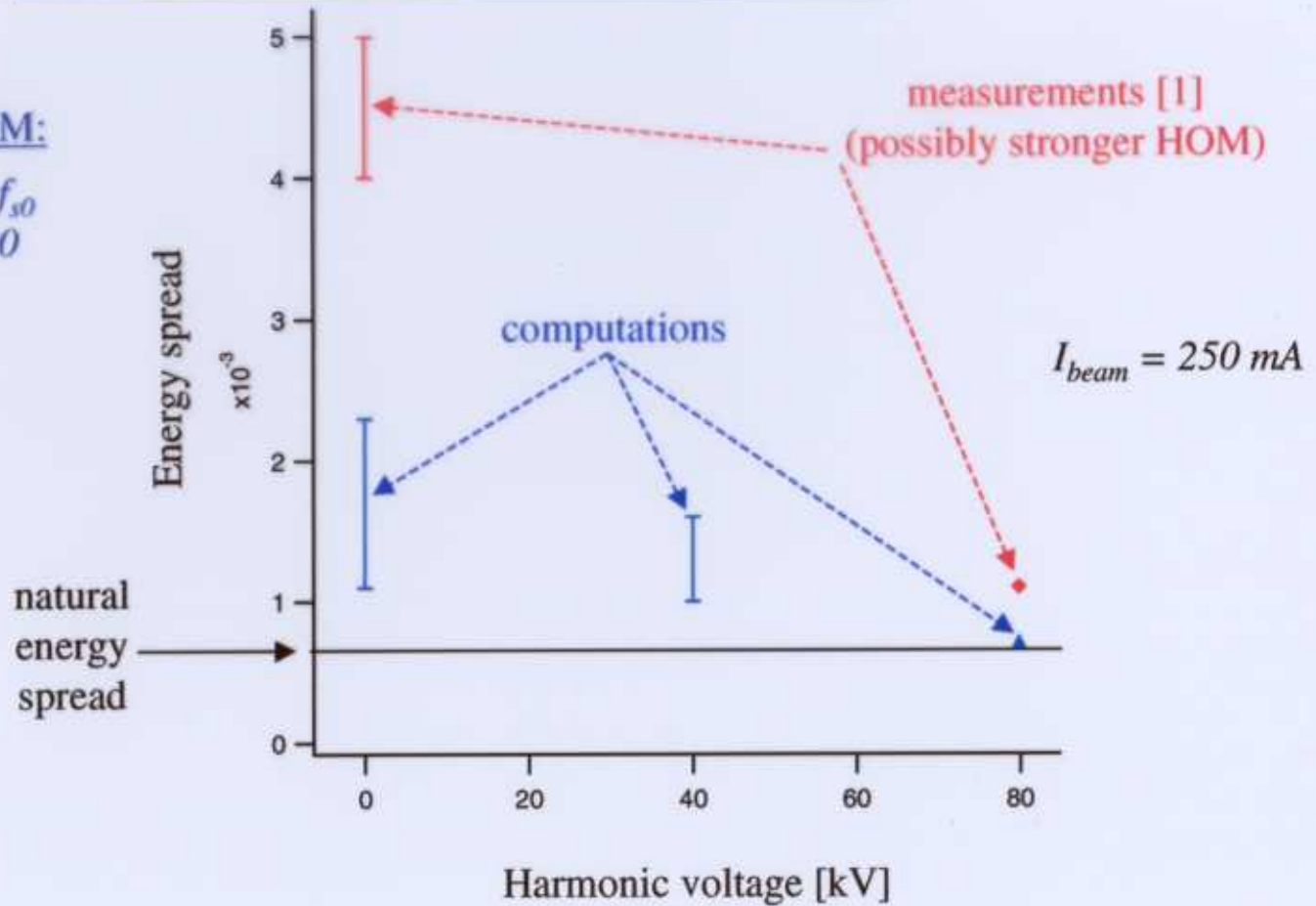
Longitudinal Coupled Bunch Instability Example of MAX-II

Arbitrarily assumed HOM:

$$f_{res,HOM} = f_{RF} + 20 \cdot f_0 + f_{s0}$$

$$R_s = 4.2 M\Omega \text{ \& } Q = 30000$$

$$I_{th} = 10 \text{ mA}$$



The new multibunch multiparticle tracking code confirms qualitatively
the observations made at MAX-II

[1] Å. Anderson & al.